Darcy Friction Factor Formulae in Turbulent Pipe Flow

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Abstract

The Darcy friction factor in turbulent pipe flow must be solved from the Colebrook equation by iteration. Because of the iteration new equations to solve this friction factor has been developed. From these equations the Blasius, Swamee-Jain and Haaland equations were validated. All three equations can be used in a smooth pipe, but the Swamee-Jain and Haaland equations are more accurate than the Blasius equation. The Blasius equation can't be used in a rough pipe. Both the Swamee-Jain and Haaland equation give good results in a rough pipe.

1 Introduction

Pressure loss in steady pipe flow is calulated using the Darcy-Weisbach equation. This equation includes the Darcy friction factor. The exact solution of the Darcy friction factor in turbulent flow is got by looking at the Moody diagram [5] or by solving it from the Colebrook equation [1].

If the Darcy friction factor must be known only once, the Moody diagram is good. This diagram is rather laborious to program in a computer code and doesn't offer any advantages.

Unfortunately the Colebrook equation must be solved by iteration. Although the solution is found usually with a few iterations, the iteration is for example in large pipe flow computing codes time consuming and a possible

*Lunowa is a small company spezialized in technical consulting. Lunowa's web-page http://www.kolumbus.fi/jukka.kiijarvi/clunowa/clunowa.html error source. The iteration needs the first guess of the value of the Darcy friction factor. How fast the iteration is, depends on how good the first guess is.

Because the boundary layer can be thin in turbulent pipe flow, the Darcy friction factor depends on the roughness of the pipe. The Colebrook equation has a term for this roughness.

In order to avoid the iteration of the Darcy friction factor from the Colebrook equation, new equations are deduced. In this study three equations to replace the Colebrook equation are validated. These three are the Blasius, Swamee-Jain [6] are Haaland [4] equation. The validation is limited only to steady turbulent pipe flow, in which the pipe is assumed to be circular. The solved Darcy friction factors are compared to the solution of the Colebrook equation.

In this study the Darcy friction factor solved from an equation is called according to the person or persons who has developed the equation. For example the Swamee-Jain friction factor means the Darcy friction factor, which is solved from the Swamee-Jain equation.

2 Equations

2.1 Darcy-Weisbach

The pressure loss in pipe flow is calculated using the known Darcy-Weisbach equation. This equation is

$$\Delta p = f \, \frac{L}{D} \, \frac{\rho V^2}{2} \tag{1}$$

where

Δp	pressure loss
f	the Darcy friction factor
L	length of pipe or pipe part
D	inner diameter of the pipe
ρ	density of fluid
V	flow velocity

2.2 Colebrook

The Colebrook equation is the most widely used equation to solve the Darcy friction factor. The Colebrook equation is [1]

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{e/D}{3.7} + \frac{2.51}{Re\sqrt{f}} \right) \tag{2}$$

in which

f	the Darcy friction factor
e	roughness of the pipe
D	inner diameter of the pipe
Re	the Reynolds number

The ratio e/D is called the relative roughness. The Reynolds number is calculated with the equation

$$Re = \frac{VD}{\nu} \tag{3}$$

where V is the flow velocity and ν the kinematic viscosity of the fluid.

The Reynolds number tells if the flow is laminar or turbulent. If the Reynolds number is smaller than the critical Reynolds number Re_{cr}, the flow is laminar. After the laminar flow regime follows the transition region. There the flow switches between laminar and turbulent randomly. When the Reynolds number reaches a certain value, the flow turns from transitional to turbulent. For pipe flow the critical Reynolds number is often assumed to be 2300. The transition region ends approximately at the Reynolds number 4000 [2].

2.3 Blasius

The Blasius equation is the most simple equation for solving the Darcy friction factor. Because the Blasius equation has no term for pipe roughness, it is valid only to smooth pipes. However, the Blasius equation is sometimes used in rough pipes because of its simplicity. The Blasius equation is valid up to the Reynolds number 10^5 [3]. The Blasius equation is

$$f = \frac{0.316}{Re^{0.25}} \tag{4}$$

where f means the Darcy friction factor and Re the Reynolds number.

2.4 Swamee-Jain

Swamee and Jain [6] have devoloped the following equation to the Darcy friction factor

$$f = 0.25 \left[\log \left(\frac{e/D}{3.7} + \frac{5.74}{Re^{0.9}} \right) \right]^{-2} \tag{5}$$

in which

f	the Darcy friction factor
e	roughness of the pipe
D	inner diameter of the pipe
Re	the Reynolds number

In the Swamee-Jain equation the Darcy friction factor is solved directly without iteration.

2.5 Haaland

Haaland [4] has deduced the equation

$$\frac{1}{f} = -1.8 \log \left[\left(\frac{e/D}{3.7} \right)^{1.11} + \frac{6.9}{Re} \right] \tag{6}$$

where

f	the Darcy friction factor
e	roughness of the pipe
D	inner diameter of the pipe
Re	the Reynolds number

In the Haaland equation there is no need to iterate the Darcy friction factor. The accuracy of the Darcy friction factor solved from this equation is claimed to be within about ±2 %, if the Reynolds number is greater than 3000 [3].

3 Code

The code for computing the Darcy friction factors was written in Scilab script language. Scilab is a free softwarre for numerical computation. It is available in the address

http://www.scilab.org/

The first guess of the Darcy friction factor in the Colebrook 2 equation is got from the Haaland equation 6. The right side of the Colebrook equation is solved with this value. After that the new friction factor on the left side is calculated. Now the right side of the Colebrook equation 2 is computed with the new Darcy friction factor. Again the left side is solved and a new Darcy friction factor got. The loop is continued until the absolute difference of the old and new friction factor is low enough.

Table 1: Computed cases. D pipe diameter, e roughness, e/D relative roughness.

D mm	e mm	e/D
1.5	0	0
1.5	0.0015	0.00100
3.0	0	0
3.0	0.0015	0.000500
6.0	0	0
6.0	0.0015	0.000250

4 Initial values

The results were computed with three pipe inner diameters, which were 1.5, 3.0 and 6.0 mm. The pipe was assumed to be drawn tubing. Then the roughness of the pipe is 0.0015 mm [5]. Additionally the Darcy friction factor was computed to a smooth pipe in each diameter. The Reynolds number was between 2300–100000 in these calculations. The computed cases are given in the table 1.

5 Results

5.1 Smooth Pipe

In the figure 1 are shown the absolute values of the Darcy friction factor solved from different equations. The pipe diameter is 1.5 mm and relative roughness 0. In one region the Darcy friction factor solved from Blasius equation is a little higher than the other friction factors. Otherwise the friction factors are almost equal. Because the friction factor in a smooth pipe is only a function of the Reynolds number, the figures of diameters 3.0 and 6.0 mm are the the same as with the diameter 1.5 mm.

In the figure 2 are given the relative errors of the Darcy friction factors solved by different equations.

The Blasius friction factor has the greatest relative error -3.5% at the Reynolds number 2300. The Swamee-Jain friction factor has the relative error 2.9 % and the Haaland friction factor 2.5% at the same Reynolds number. Then the Blasius friction factor has the greatest relative error up to the Reynolds number about 85000. The absolute value of the relative error of the Blasius friction factor increases after the Reynolds number 100000,

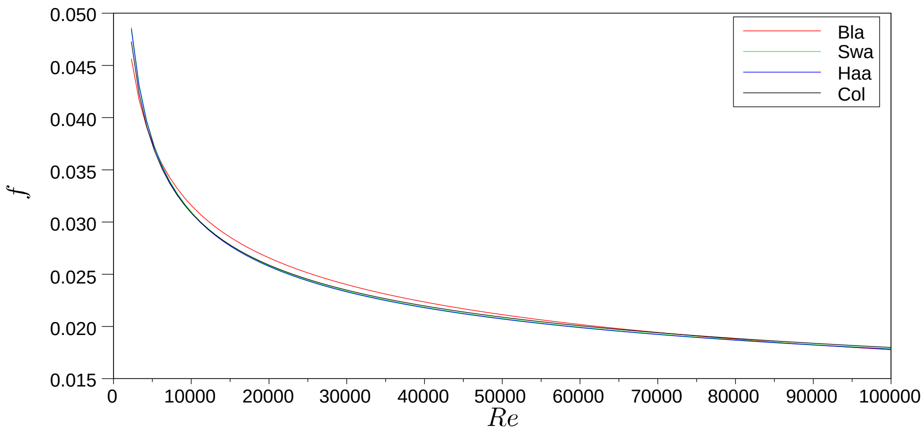

Figure 1: The Darcy friction factor f solved from different equations as a function of the Reynolds number Re. Bla Blasius equation 4, Swa Swamee-Jain equation 5, Haa Haaland equation 6 and Col Colebrook equation 2. $D = 1.5$ mm, $e = 0$, $e/D = 0$.

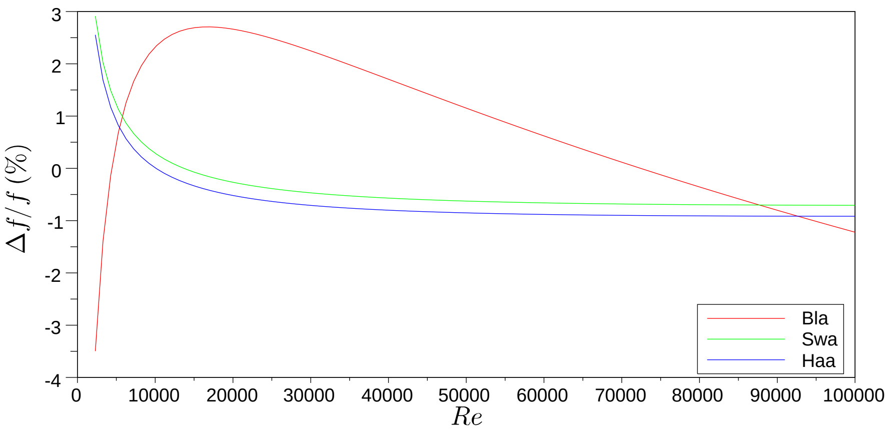

Figure 2: Relative errors $\Delta f/f$ of the Darcy friction factor solved from different equations as a function of the Reynolds number Re. Bla Blasius equation 4, Swa Swamee-Jain equation 5 and Haa Haaland equation 6. $D = 1.5$ mm, $e = 0$, $e/D = 0$.

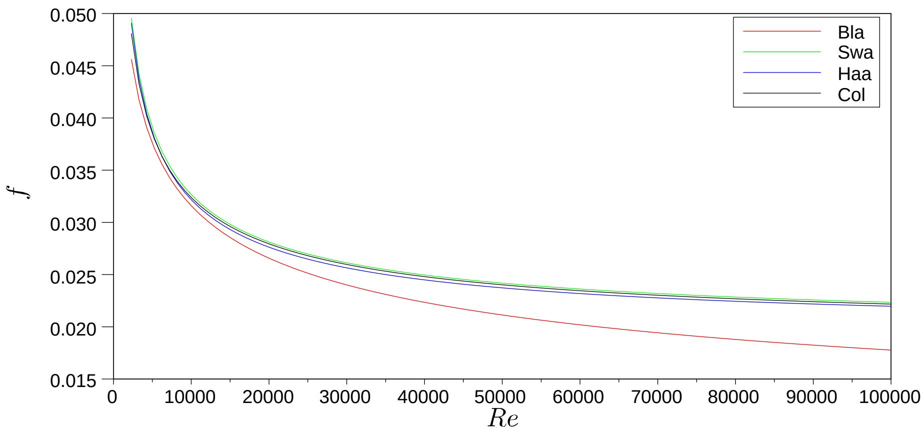

Figure 3: The Darcy friction factor f solved from different equations as a function of the Reynolds number Re. Bla Blasius equation 4, Swa Swamee-Jain equation 5, Haa Haaland equation 6 and Col Colebrook equation 2. $D = 1.5$ mm, $e = 0.0015$ mm, $e/D = 0.00100$.

but it is not shown in the figure 2. The relative errors of the Swamee-Jain and Haaland friction factors are approximately constant after the Reynolds number 60000. Then the relative error of the Swamee-Jain friction factor is -0.7 % and of the Haaland friction factor -0.9%.

5.2 Rough Pipe

The Darcy friction factors solved from different equations are shown in the figures 3, 4 and 5. There can't be seen no significant difference in the friction factors solved from the Swamee-Jain, Haaland and Colebrook equation. The Blasius friction factor differs from the others. When the relative roughness gets smaller, the Blasius friction factor differs from the other friction factors less.

The relative errors of the Darcy friction factors solved from different equations are given in the figures 6, 7 and 8.

When the Reynolds number is 2300, the relative error gets smaller in every case when the relative roughness decreases. The relative error of the Blasius friction factor is usually the greatest. At the small Reynolds numbers, the error of the Haaland friction factor is smaller than the error of the Swamee-Jain friction factor. Else the error of the Swamee-Jain friction factor is the smallest. When the pipe diameter is 6.0 mm, the relative error of the Swamee-Jain friction factor is only 0.05 % at the Reynolds number 100000.

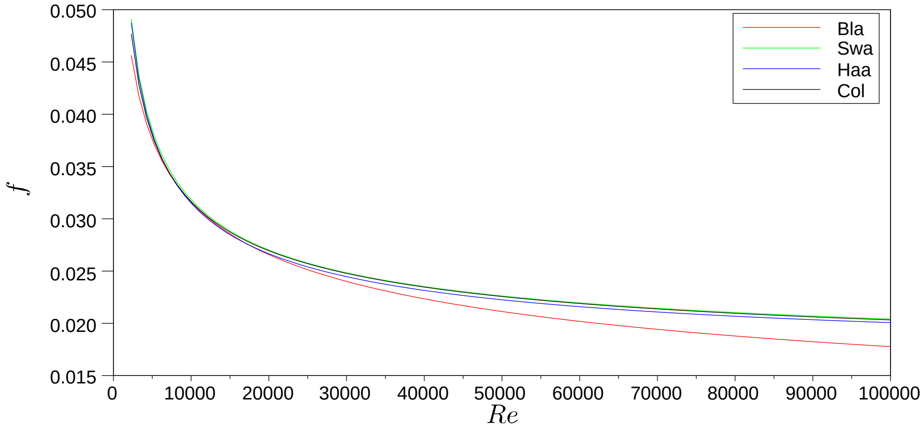

Figure 4: The Darcy friction factor f solved from different equations as a function of the Reynolds number Re. Bla Blasius equation 4, Swa Swamee-Jain equation 5, Haa Haaland equation 6 and Col Colebrook equation 2. $D = 3.0$ mm, $e = 0.0015$ mm, $e/D = 0.00050$.

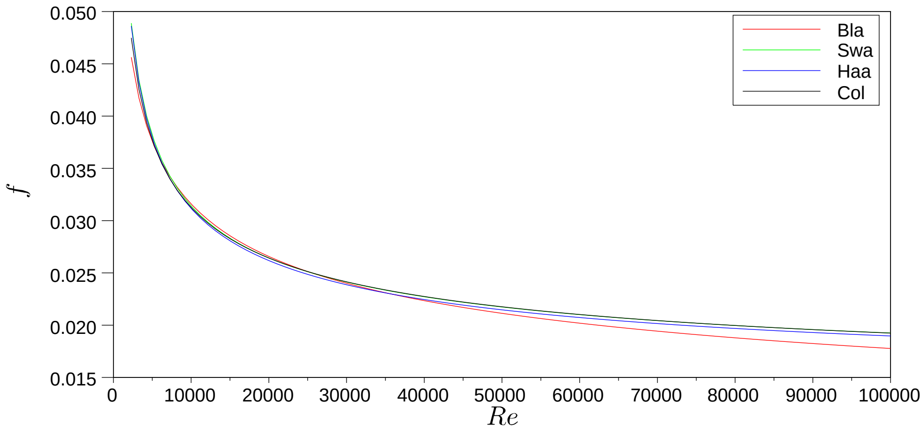

Figure 5: The Darcy friction factor f solved from different equations as a function of the Reynolds number Re. Bla Blasius equation 4, Swa Swamee-Jain equation 5, Haa Haaland equation 6 and Col Colebrook equation 2. $D = 6.0$ mm, $e = 0.0015$ mm, $e/D = 0.00025$.

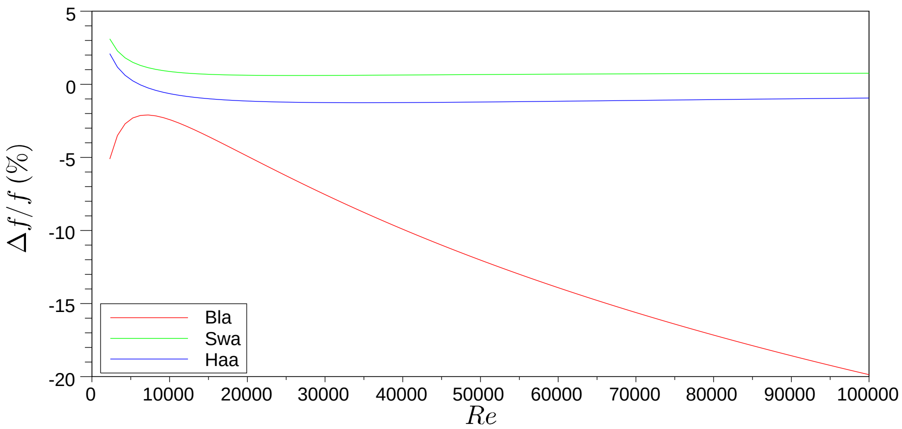

Figure 6: The relative error $\Delta f/f$ of the Darcy friction factor solved from different equations as a function of the Reynolds number Re. Bla Blasius equation 4, Swa Swamee-Jain equation 5, Haa Haaland equation 6 and Col Colebrook equation 2. $D = 1.5$ mm, $e = 0.0015$ mm, $e/D = 0.00100$.

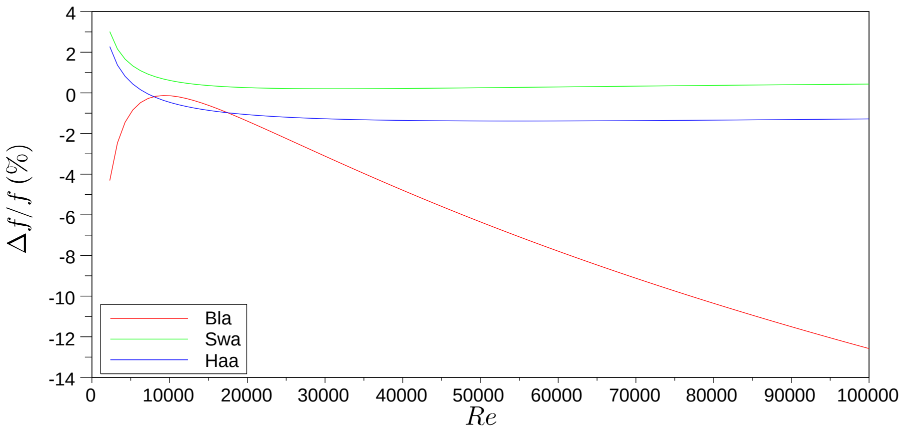

Figure 7: The relative error $\Delta f/f$ of the Darcy friction factor solved from different equations as a function of the Reynolds number Re. Bla Blasius equation 4, Swa Swamee-Jain equation 5, Haa Haaland equation 6 and Col Colebrook equation 2. $D = 3.0$ mm, $e = 0.0015$ mm, $e/D = 0.00050$.

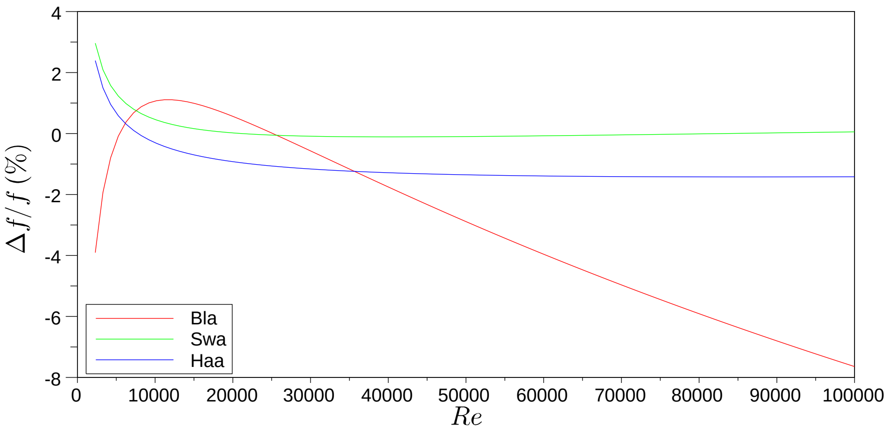

Figure 8: The relative error $\Delta f/f$ of the Darcy friction factor solved from different equations as a function of the Reynolds number Re. Bla Blasius equation 4, Swa Swamee-Jain equation 5, Haa Haaland equation 6 and Col Colebrook equation 2. $D = 6.0$ mm, $e = 0.0015$ mm, $e/D = 0.00025$.

6 Discussion

In this study the Darcy friction factor was solved from four different equations in turbulent pipe flow. The results were computed in a smooth pipe and with three relative rougnesses of the pipe.

The solution of the Colebrook equation was assumed to be the exact solution. The other solved Darcy friction factors were compared to the solution of the Colebrook equation.

During the transitional flow the greatest relative errors of the solutions of the Swamee-Jain and Haaland equations were found. The solution of the Blasius equation was great in this region, too. There are large uncertainties in the Darcy friction factor in this flow regime.

In a smooth pipe the solution of the Darcy friction factor from the Blasius equation is accurate enough in many applications. In these cases the Reynolds number must be smaller than 100000. Both the Swamee-Jain and Haaland equation are more accurate than the Blasius equation to solve the Darcy friction factor in a smooth pipe.

The Blasius equation can't replace the Colebrook, Swamee-Jain and Haaland equation, when the Darcy friction factor must be solved in a rough pipe. The Blasius equation is appropriate only in a smooth pipe.

Both the Swamee-Jain and Haaland equation give a good approximation of the Darcy friction factor. Based on the computed results it can't be said, which one these two equations gives the better solution of the Darcy friction factor. The Swamee-Jain and Haaland equation can be used in most cases instead of the Colebrook equation to solve the Darcy friction factor.

References

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